Eur. Phys. J. C 14, 91–101 (2000)
Digital Object Identifier (DOI) 10.1007/s100520000310

THE EUROPEAN
PHYSICAL JOURNAL C

Yukawa corrections to the charged Higgs boson production in association with the top quark at hadron colliders

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Received: 12 October 1999 / Revised version: 3 December 1999 /
Published online: 6 April 2000 – © Springer-Verlag 2000

Abstract. We calculate the Yukawa corrections of order $O(\alpha_{\rm ew} m_{t(b)}^2/m_W^2)$ to charged Higgs boson production in association with a top quark at the Tevatron and the LHC. The corrections are not very sensitive to the mass of the charged Higgs boson and can exceed -20% for low values of $\tan\beta$, where the contribution of the top quark is large, and high values of $\tan\beta$ where the contribution of the bottom quark becomes large. These Yukawa corrections could be significant for charged Higgs boson searches based on this production process, particularly at the LHC where the cross section is relatively large.

1 Introduction

There has been a great deal of interest in the charged Higgs bosons appearing in the two-Higgs-double model (THDM) [1], particularly the minimal supersymmetric standard model (MSSM) [2], which predicts the existence of three neutral and two charged Higgs bosons h, H, A, and $H^\pm$. The lightest neutral Higgs boson may be difficult to distinguish from the neutral Higgs boson of the standard model (SM), but charged Higgs bosons carry a distinctive signature of the Higgs sector in the MSSM. Therefore, the search for charged Higgs bosons is very important for probing the Higgs sector of the MSSM and, therefore, will be one of the prime objectives of the CERN Large Hadron Collider (LHC). At the LHC the integrated luminosity is expected to reach $L = 100\,\mathrm{fb}^{-1}$ within several years. Recently, several studies of charged Higgs boson production at hadron colliders have appeared in the literature [3–5]. For a relatively light charged Higgs boson, $m_{H^\pm} < m_t - m_b$, the dominant production processes at the LHC are $gg, q\bar{q} \to t\bar{t}$ followed by the decay sequence $t \to bH^+ \to b\tau^+\nu_\tau$ [6]. For a heavier charged Higgs boson the dominant production process is $gb \to tH^-$ [7–9]. Previous studies showed that the search for heavy charged Higgs bosons with $m_{H^+} > m_t + m_b$ at a hadron collider is seriously complicated by QCD backgrounds due to processes such as $gb \to t\bar{t}b, g\bar{b} \to t\bar{t}\bar{b}$, and $gg \to t\bar{t}b\bar{b}$, as well as other processes [8]. However, recent analyses [10,11] indicate that the decay mode $H^+ \to \tau^+\nu$ provides an excellent signature for a heavy charged Higgs boson in searches at the LHC. The discovery region for $H^\pm$ is far greater than had been thought for a large range of the $(m_{H^\pm}, \tan\beta)$ parameter space, extending beyond $m_{H^\pm} \sim 1\,\mathrm{TeV}$ and down to at least $\tan\beta \sim 3$, and potentially to $\tan\beta \sim 1.5$, assuming the latest results for the SM parameters and parton distribution functions as well as using kinematic selection techniques and the tau polarization analysis [11].

The one-loop radiative corrections to H^-t associated production have not been calculated, although this production process has been studied extensively at tree level [7–9]. In this paper we present the calculations of the Yukawa corrections to this associated H^-t production process at both the Fermilab Tevatron and the LHC in the MSSM. These corrections arise from the virtual effects of the third family (top and bottom) quarks, the charged and neutral Higgs bosons, as well as the Goldstone bosons. In order to get a reliable estimate this process has to be merged with the related gluon splitting contribution $gg \to H^-t\bar{b}$. This leads to a suppression by about 50% at LO [12]. However, the complete one-loop QCD corrections are probably more important, but not yet available.

2 Calculation

The tree level amplitude for $gb \to tH^-$ is

$$M_0 = M_0^{(s)} + M_0^{(t)}, \tag{1}$$

where $M_0^{(s)}, M_0^{(t)}$ represent the amplitudes arising from diagrams in Figs. 1a,b, respectively. Explicitly,

$$M_0^{(s)} = \frac{\mathrm{i} g g_s}{\sqrt{2} m_W (\hat{s} - m_b^2)} \overline{u}(p_t)[2m_t \cot\beta p_b^\mu P_{\rm L}$$

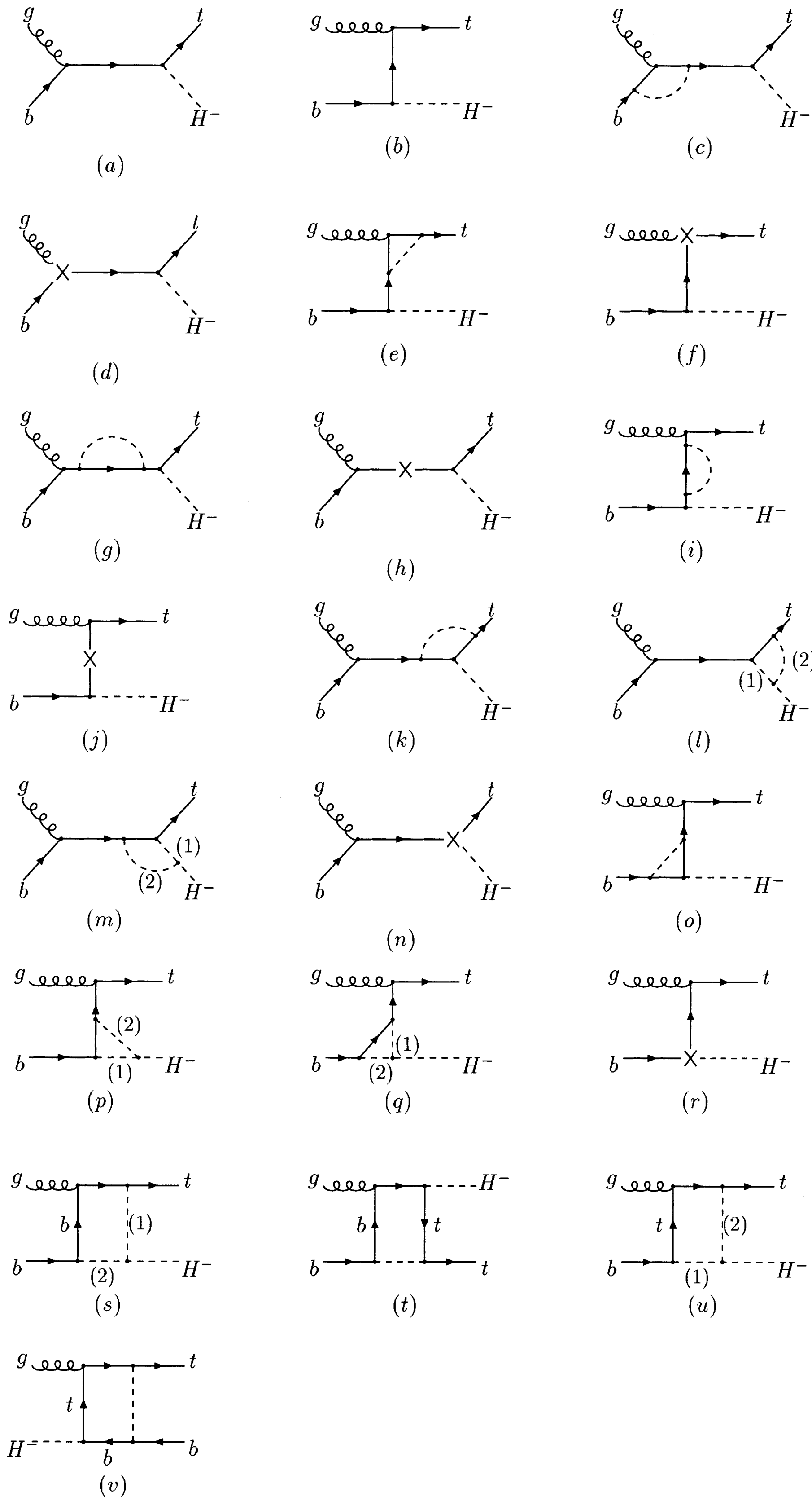


Fig. 1a–u. Feynman diagrams contributing to the $O(\alpha_{\rm ew} m_{t(b)}^2/m_W^2)$ Yukawa corrections to $gb \to tH^-$: **a** and **b** are the tree level diagrams; **c** and **e** are gqq ($q = b, t$) vertex diagrams; **g** and **i** are self-energy diagrams; **k**–**m** and **o**–**q** are gbH^- vertices; **s**–**v** are box diagrams; **d**, **f**, **h**, **j**, **n** and **r** are counterterm diagrams. The dashed lines represent H, h, A, $H^\pm$, G^0, $G^\pm$ for diagrams **c**, **e**, **g** and **i**, and H, h, A, G^0 for diagrams **k**, **o**, **t** and **v**. For diagrams **l**, **m**, **p**, **q**, **s** and **u**, the dashed line (2) represents H and h when the dashed line (1) is H^-, and H, h and A when the line (1) is G^-

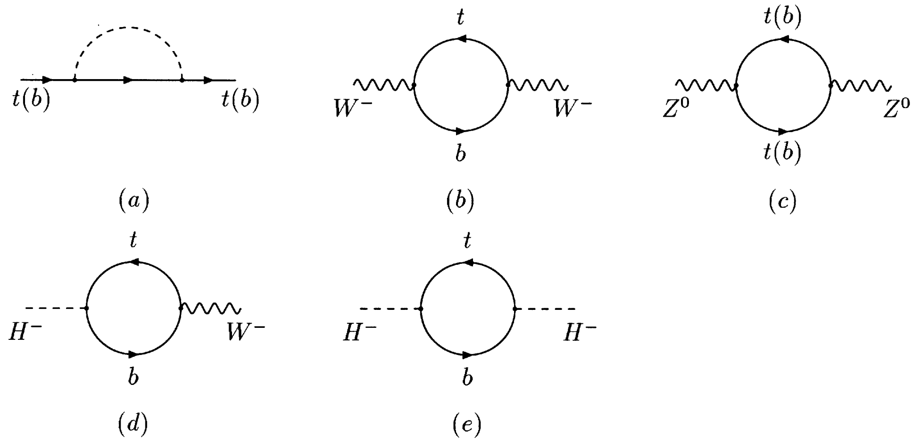


Fig. 2a–e. Self-energy Feynman diagrams contributing to renormalization constants. The dashed lines represent H, h, A, $H^{\pm}$, G^0, $G^{\pm}$ in **a**

$$
\begin{aligned}
&+ 2m_b \tan\beta p_b^\mu P_R - m_t \cot\beta \gamma^\mu \not{k} P_{\rm L} \\
&- m_b \tan\beta \gamma^\mu \not{k} P_R] u(p_b) \varepsilon_\mu(k) T_{ij}^a, \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
M_0^{(t)} &= \frac{\mathrm{i} g g_s}{\sqrt{2} m_W (\hat{t} - m_t^2)} \overline{u}(p_t)[2m_t \cot\beta p_t^\mu P_{\rm L} \\
&+ 2m_b \tan\beta p_t^\mu P_R - m_t \cot\beta \gamma^\mu \not{k} P_{\rm L} \\
&- m_b \tan\beta \gamma^\mu \not{k} P_R] u(p_b) \varepsilon_\mu(k) T_{ij}^a, \qquad (3)
\end{aligned}
$$

where T^a are the $SU(3)$ color matrices and $\hat{s}$ and $\hat{t}$ are the subprocess Mandelstam variables defined by

$$\hat{s} = (p_b + k)^2 = (p_t + p_{H^-})^2,$$

and

$$\hat{t} = (p_t - k)^2 = (p_{H^-} - p_b)^2.$$

Here the Cabbibo–Kobayashi–Maskawa matrix element $V_{\rm CKM}[bt]$ has been taken to be unity.

The Yukawa corrections of order $O(\alpha_{\rm ew} m_{t(b)}^2/m_W^2)$ to the process $gb \to H^- t$ arise from the Feynman diagrams shown in Figs. 1c–v and Fig. 2. We carried out the calculation in the 't Hooft–Feynman gauge and used dimensional regularization to regulate all the ultraviolet divergences in the virtual-loop corrections using the on-mass-shell renormalization scheme [13], in which the fine-structure constant $\alpha_{\rm ew}$ and physical masses are chosen to be the renormalized parameters, and finite parts of the counterterms are fixed by the renormalization conditions. The coupling constant g is related to the input parameters e, m_W, and m_Z by $g^2 = e^2/s_w^2$, and $s_w^2 = 1 - m_w^2/m_Z^2$. The parameter β in the MSSM we are considering must also be renormalized. Following the analysis of [14], this renormalization constant was fixed by the requirement that the on-mass-shell $H^+ \bar{l} \nu_l$ coupling remains of the same form as in (2) of [14] to all orders of perturbation theory. Taking into account the $O(\alpha_{\rm ew} m_{t(b)}^2/m_W^2)$ Yukawa corrections, the renormalized amplitude for the process $gb \to tH^-$ can be written as

$$
\begin{aligned}
M_{ren} &= M_0^{(s)} + M_0^{(t)} + \delta M^{V_1(s)} + \delta M^{V_1(t)} + \delta M^{s(s)} \\
&+ \delta M^{s(t)} + \delta M^{V_2(s)} + \delta M^{V_2(t)} + \delta M^{b(s)} + \delta M^{b(t)} \\
&\equiv M_0^{(s)} + M_0^{(t)} + \sum_l \delta M^l, \qquad (4)
\end{aligned}
$$

where $\delta M^{V_1(s)}, \delta M^{V_1(t)}, \delta M^{s(s)}, \delta M^{s(t)}, \delta M^{V_2(s)}, \delta M^{V_2(t)}$, $\delta M^{b(s)}$ and $\delta M^{b(t)}$ represent the corrections to the tree diagrams arising, respectively, from the gbb vertex diagram Fig. 1c, the gtt vertex diagram Fig. 1e, the bottom quark self-energy diagram Fig. 1g, the top quark self-energy diagram Fig. 1i, the $\bar{b}tH^-$ vertex diagrams Figs. 1k–m and Figs. 1o–q, including their corresponding counterterms Fig. 1d,f,h,j,n,r, and the box diagrams Figs. 1s–v. $\sum_l \delta M^l$ then represents the sum of the contributions to the Yukawa corrections from all the diagrams in Figs. 1c–v. The explicit form of δM^l can be expressed as

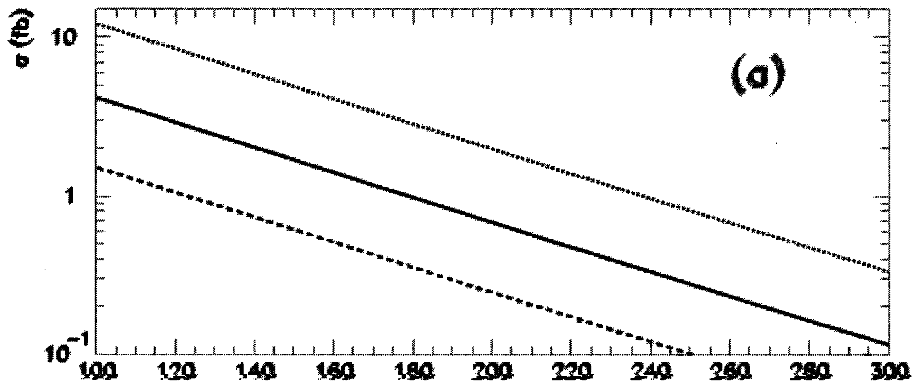
$$
\begin{aligned}
\delta M^l &= -\frac{\mathrm{i} g^3 g_s}{4\sqrt{2} \times 16\pi^2 m_W} C^l \overline{u}(p_t) \{ f_1^l \gamma^\mu P_{\rm L} + f_2^l \gamma^\mu P_R \\
&+ f_3^l p_b^\mu P_{\rm L} + f_4^l p_b^\mu P_R + f_5^l p_t^\mu P_{\rm L} + f_6^l p_t^\mu P_R \\
&+ f_7^l \gamma^\mu \not{k} P_{\rm L} + f_8^l \gamma^\mu \not{k} P_R + f_9^l p_b^\mu \not{k} P_{\rm L} + f_{10}^l p_b^\mu \not{k} P_R \\
&+ f_{11}^l p_t^\mu \not{k} P_{\rm L} + f_{12}^l p_t^\mu \not{k} P_R \} u(p_b) \varepsilon_\mu(k) T_{ij}^a, \qquad (5)
\end{aligned}
$$

where the C^l are coefficients that depend on $\hat{s}, \hat{t}$, and the masses, and the f_i^l are the form factors; both the coefficients C^l and the form factors f_i^l are given explicitly in Appendix A. The corresponding amplitude squared is

$$
\begin{aligned}
\overline{\sum} |M_{\rm ren}|^2 &= \overline{\sum} |M_0^{(s)} + M_0^{(t)}|^2 \\
&+ 2\mathrm{Re} \overline{\sum} \left[\left(\sum_l \delta M^l \right) (M_0^{(s)} + M_0^{(t)})^\dagger \right], \qquad (6)
\end{aligned}
$$

where

$$
\begin{aligned}
\overline{\sum} |M_0^{(s)} + M_0^{(t)}|^2 &= \frac{g^2 g_s^2}{2 N_C m_W^2} \Bigg\{ \frac{1}{(\hat{s} - m_b^2)^2} [(m_t^2 \cot^2\beta \\
&+ m_b^2 \tan^2\beta)(p_b \cdot k p_t \cdot k - m_b^2 p_t \cdot k \\
&+ 2 p_b \cdot k p_b \cdot p_t - m_b^2 p_b \cdot p_t) \\
&+ 2 m_b^2 m_t^2 (p_b \cdot k - m_b^2)] + \frac{1}{(\hat{t} - m_t^2)^2} [(m_t^2 \cot^2\beta \\
&+ m_b^2 \tan^2\beta)(p_b \cdot k p_t \cdot k + m_t^2 p_b \cdot k - m_t^2 p_b \cdot p_t) \\
&+ 2 m_b^2 m_t^2 (p_t \cdot k - m_t^2)] + \frac{1}{(\hat{s} - m_b^2)(\hat{t} - m_t^2)}
\end{aligned}
$$



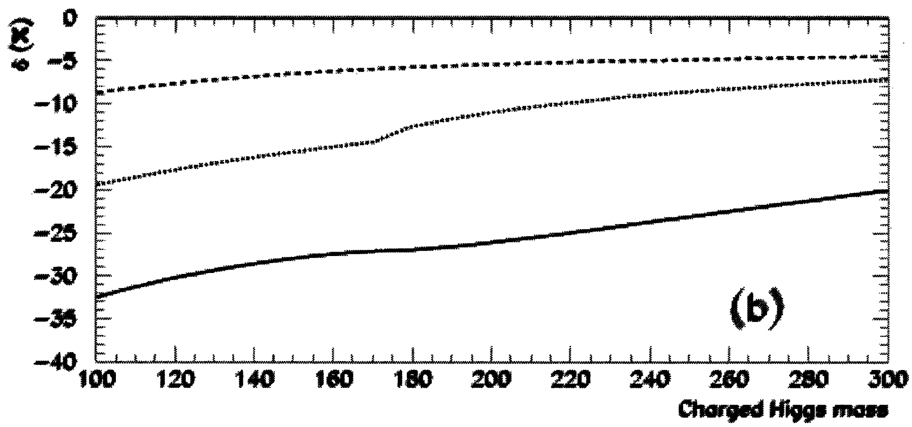


Fig. 3a,b. The tree level total cross sections **a** and relative one-loop Yukawa corrections **b** versus $m_{H^\pm}$ at the Tevatron with $s^{1/2} = 2\,\mathrm{TeV}$. The solid, dashed and dotted lines correspond to $\tan\beta = 2$, 10 and 30, respectively

$$\times[(m_t^2\cot^2\beta + m_b^2\tan^2\beta)(2p_b\cdot kp_t\cdot k + 2p_b\cdot kp_b\cdot p_t - 2(p_b\cdot p_t)^2 - m_b^2 p_t\cdot k + m_t^2 p_b\cdot k) + 2m_b^2 m_t^2(p_t\cdot k - p_b\cdot k - 2p_b\cdot p_t)]\Big\}, \quad (7)$$

$$\overline{\sum}\delta M^i(M_0^{(s)})^\dagger = -\frac{g^4 g_s^2}{64N_C\times 16\pi^2 m_W^2(\hat{s}-m_b^2)} \times C^l\sum_{i=1}^{12} h_i^{(s)} f_i^l, \quad (8)$$

and

$$\overline{\sum}\delta M^i(M_0^{(t)})^\dagger = -\frac{g^4 g_s^2}{64N_C\times 16\pi^2 m_W^2(\hat{t}-m_t^2)} \times C^l\sum_{i=1}^{12} h_i^{(t)} f_i^l. \quad (9)$$

Here the color factor $N_C = 3$ and $h_i^{(s)}, h_i^{(t)}$ are scalar functions whose explicit expressions are given in Appendix B.

The cross section for the process $gb \to tH^-$ is

$$\hat{\sigma} = \int_{\hat{t}_{\min}}^{\hat{t}_{\max}} \frac{1}{16\pi\hat{s}^2}\overline{\Sigma}|M_{\mathrm{ren}}|^2\mathrm{d}\hat{t} \quad (10)$$

with

$$\hat{t}_{\min} = \frac{m_t^2 + m_{H^-}^2 - \hat{s}}{2} - \frac{1}{2}\sqrt{(\hat{s}-(m_t+m_{H^-})^2)(\hat{s}-(m_t-m_{H^-})^2)},$$

and

$$\hat{t}_{\max} = \frac{m_t^2 + m_{H^-}^2 - \hat{s}}{2} + \frac{1}{2}\sqrt{(\hat{s}-(m_t+m_{H^-})^2)(\hat{s}-(m_t-m_{H^-})^2)}.$$

The total hadronic cross section for $pp \to gb \to tH^-$ can be obtained by folding the subprocess cross section $\hat{\sigma}$ with the parton luminosity:

$$\sigma(s) = \int_{\tau_0}^{1}\frac{\mathrm{d}\tau}{\tau}\left(\frac{1}{s}\frac{\mathrm{d}L}{\mathrm{d}\tau}\right)(\hat{s}\hat{\sigma})(gb\to tH^- \text{ at } \hat{s}=\tau s), \quad (11)$$

where $\tau_0 = (m_t+m_{H^-})^2/s$, $s^{1/2}$ and $\hat{s}^{1/2}$ are the CM energies of the pp and gb states, respectively, and the quantity $\mathrm{d}L/\mathrm{d}\tau$ is the parton luminosity, which is defined as

$$\frac{\mathrm{d}L}{\mathrm{d}\tau} = \int_{\tau}^{1}\frac{\mathrm{d}x}{x}f_{b/P}(x,q)f_{g/P}(\tau/x,q), \quad (12)$$

where $f_{b/P}(x,q)$ and $f_{g/P}(\tau/x,q)$ are the bottom quark and gluon parton distribution functions.

3 Numerical results

In the following we present some numerical results for charged Higgs boson production in association with a top

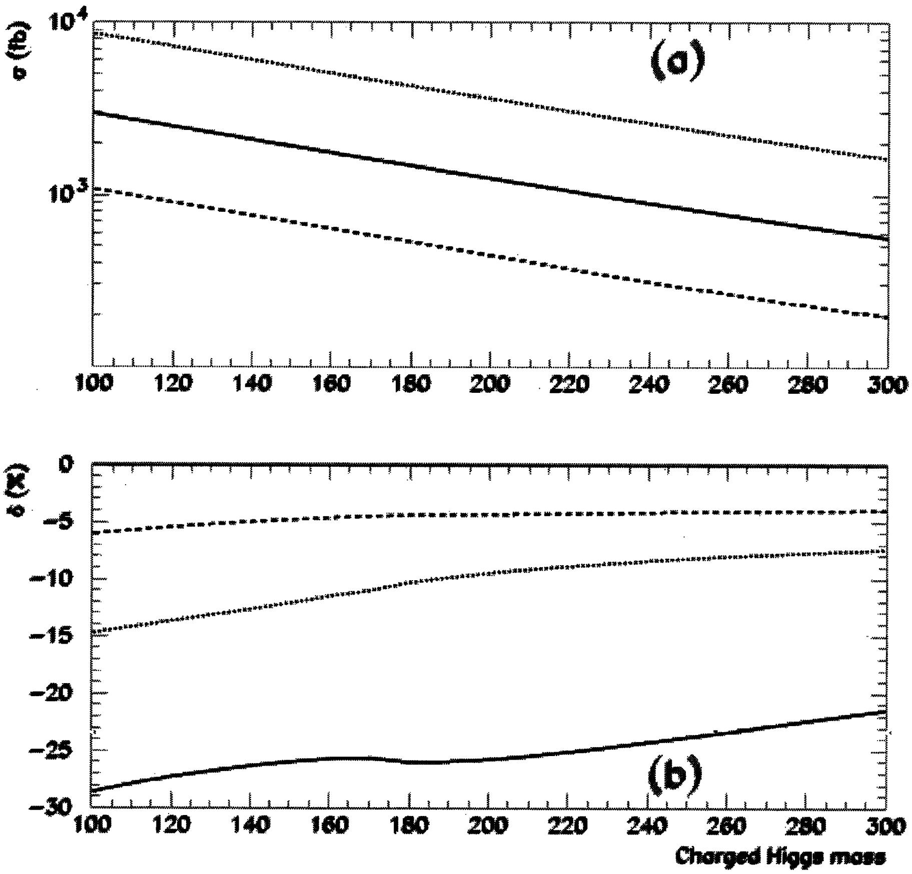


Fig. 4a,b. The tree level total cross sections **a** and relative one-loop Yukawa corrections **b** versus $m_{H^\pm}$ at the LHC with $s^{1/2} = 14\,\mathrm{TeV}$. The solid, dashed and dotted lines correspond to $\tan\beta =$ 2, 10 and 30, respectively

quark at both the Tevatron and the LHC. In our numerical calculations the SM parameters were taken to be $m_W = 80.41\,\mathrm{GeV}$, $m_Z = 91.187\,\mathrm{GeV}$, $m_t = 175\,\mathrm{GeV}$, $m_b = 4.4\,\mathrm{GeV}$, $\alpha_s(m_Z) = 0.119$, and $\alpha_{\mathrm{ew}}(m_Z) = 1/128.8$ [15]. We also used the one-loop relations [16] from the MSSM between the Higgs boson masses $m_{h,H,A,H^\pm}$ and the parameters α and β, and chose $m_{H^\pm}$ and $\tan\beta$ as the two independent input parameters. And we used the CTEQ5M [17] parton distributions throughout the calculations.

Figures 3a and 4a show the tree level total cross sections as a function of the charged Higgs boson mass for three representative values of $\tan\beta$. For $m_{H^\pm} = 200\,\mathrm{GeV}$ the total cross sections at the Tevatron are at most only a few fb for $\tan\beta = 2$, 10, and 30, and for $m_{H^\pm} = 300\,\mathrm{GeV}$ the total cross sections are smaller than 1 fb for all three values of $\tan\beta$. However, at the LHC the total cross sections are much larger: the order of thousands of fb for $m_{H^\pm}$ in the range 100 to 300 GeV and $\tan\beta = 2$ and 30; and they are hundreds of fb for the intermediate value $\tan\beta = 10$. For low $\tan\beta$ the top quark contribution is enhanced while for high $\tan\beta$ the bottom quark contribution becomes large. These results agree with [8,9] and, it should be noted, are larger than the $W^\pm H^\pm$ associated production cross section at the LHC [4].

In Figs. 3b and 4b we show the corrections to the total cross sections relative to the tree level values as a function of $m_{H^\pm}$ for $\tan\beta = 2, 10$, and 30. These corrections decrease the total cross sections significantly for a wide range of the charged Higgs boson mass, especially for the smaller values of $\tan\beta$ where the top quark contribution is greatly enhanced. In particular, for $\tan\beta = 2$ the corrections exceed -20% for $m_{H^\pm}$ below 300 GeV and reach more than -25% for $m_{H^\pm}$ below 200 GeV at both the Tevatron and the LHC.

In conclusion, we have calculated the Yukawa corrections of order $O(\alpha_{\mathrm{ew}} m_{t(b)}^2/m_W^2)$ to the cross section for charged Higgs boson production in association with a top quark at the Tevatron and the LHC. These corrections decrease the cross section and are not very sensitive to the mass of the charged Higgs boson, but depend more strongly on $\tan\beta$. At low $\tan\beta$ the top quark contribution is enhanced while at high $\tan\beta$ the bottom quark contribution becomes large. For $m_{H^\pm}$ in the range 100 to 300 *GeV* the Yukawa corrections are as large as -30% for $\tan\beta = 2$, then become smaller for the intermediate value $\tan\beta = 10$, but increase to nearly -20% for $\tan\beta = 30$.

Acknowledgements. This work was supported in part by the National Natural Science Foundation of China, the Doctoral Program Foundation of Higher Education of China, the Post Doctoral Foundation of China, a grant from the State Commission of Science and Technology of China, and the U.S. Department of Energy, Division of High Energy Physics, under Grant No.DE-FG02-91-ER4086. S.H. Zhu also gratefully acknowledges the support of the K.C. Wong Education Foundation of Hong Kong.

Appendix A

The coefficients C^l and form factors f_i^l are the following:

$$C^{V_1(s)} = \frac{m_b^2}{m_W^2(\hat{s} - m_b^2)}, \quad C^{V_1(t)} = \frac{m_t^2}{m_W^2(\hat{t} - m_t^2)},$$
$$C^{s(s)} = \frac{m_b^2}{m_W^2(\hat{s} - m_b^2)^2}, \quad C^{s(t)} = \frac{m_t^2}{m_W^2(\hat{t} - m_t^2)^2},$$
$$C^{V_2(s)} = \frac{1}{\hat{s} - m_b^2}, \quad C^{V_2(t)} = \frac{1}{\hat{t} - m_t^2},$$
$$C^{b(s)} = C^{b(t)} = \frac{1}{m_W},$$
$$f_1^{V_1(s)} = \eta^{(1)}[m_b(g_2^{V_1(s)} - g_3^{V_1(s)}) - 2p_b \cdot k g_6^{V_1(s)}],$$
$$f_2^{V_1(s)} = \eta^{(2)}[m_b(g_3^{V_1(s)} - g_2^{V_1(s)}) - 2p_b \cdot k g_7^{V_1(s)}],$$
$$f_3^{V_1(s)} = \eta^{(2)}[2(g_1^{V_1(s)} + g_2^{V_1(s)}) + m_b(g_4^{V_1(s)} + g_5^{V_1(s)}) + 2p_b \cdot k g_8^{V_1(s)}],$$
$$f_4^{V_1(s)} = \eta^{(1)}[2(g_1^{V_1(s)} + g_3^{V_1(s)}) + m_b(g_4^{V_1(s)} + g_5^{V_1(s)}) + 2p_b \cdot k g_9^{V_1(s)}],$$
$$f_7^{V_1(s)} = \eta^{(2)}[-(g_1^{V_1(s)} + g_2^{V_1(s)}) + m_b(g_6^{V_1(s)} + g_7^{V_1(s)})],$$
$$f_8^{V_1(s)} = \eta^{(1)}[-(g_1^{V_1(s)} + g_3^{V_1(s)}) + m_b(g_6^{V_1(s)} + g_7^{V_1(s)})],$$
$$f_9^{V_1(s)} = \eta^{(1)}[g_4^{V_1(s)} + 2g_6^{V_1(s)} + m_b(g_8^{V_1(s)} - g_9^{V_1(s)})],$$
$$f_{10}^{V_1(s)} = \eta^{(2)}[g_5^{V_1(s)} + 2g_7^{V_1(s)} + m_b(g_9^{V_1(s)} - g_8^{V_1(s)})],$$
$$f_1^{V_2(s)} = 2p_b \cdot k g_3^{V_2(s)}, \quad f_2^{V_2(s)} = 2p_b \cdot k g_4^{V_2(s)},$$
$$f_3^{V_2(s)} = 2g_1^{V_2(s)} + 2m_t \cot\beta \delta \Lambda_{\mathrm{L}}^{btH^-} - 2m_t g_3^{V_2(s)} + 2m_b g_4^{V_2(s)},$$
$$f_4^{V_2(s)} = 2g_2^{V_2(s)} + 2m_b \tan\beta \delta \Lambda_R^{btH^-} + 2m_b g_3^{V_2(s)} - 2m_t g_4^{V_2(s)},$$
$$f_7^{V_2(s)} = -\frac{1}{2} f_3^{V_2(s)}, \quad f_8^{V_2(s)} = -\frac{1}{2} f_4^{V_2(s)},$$
$$f_1^{V_2(t)} = 2p_t \cdot k g_3^{V_2(t)}, \quad f_2^{V_2(t)} = 2p_t \cdot k g_4^{V_2(t)},$$
$$f_5^{V_2(t)} = 2g_1^{V_2(t)} + 2m_t \cot\beta \delta \Lambda_{\mathrm{L}}^{btH^-} - 2m_t g_3^{V_2(t)} + 2m_b g_4^{V_2(t)},$$
$$f_6^{V_2(t)} = 2g_2^{V_2(t)} + 2m_b \tan\beta \delta \Lambda_R^{btH^-} + 2m_b g_3^{V_2(t)} - 2m_t g_4^{V_2(t)},$$
$$f_7^{V_2(t)} = -\frac{1}{2} f_5^{V_2(t)}, \quad f_8^{V_2(t)} = -\frac{1}{2} f_6^{V_2(t)},$$
$$f_1^{s(s)} = 2\eta^{(1)} p_b \cdot k[g_1^{s(s)} + m_b(g_2^{s(s)} + g_3^{s(s)})],$$
$$f_2^{s(s)} = 2\eta^{(2)} p_b \cdot k[g_1^{s(s)} + m_b(g_2^{s(s)} + g_4^{s(s)})],$$
$$f_3^{s(s)} = 2\eta^{(2)}[2m_b g_1^{s(s)} + 2(m_b^2 + p_b \cdot k) g_2^{s(s)} + (m_b^2 + 2p_b \cdot k) g_3^{s(s)} + m_b^2 g_4^{s(s)}],$$
$$f_4^{s(s)} = 2\eta^{(1)}[2m_b g_1^{s(s)} + 2(m_b^2 + p_b \cdot k) g_2^{s(s)} + m_b^2 g_3^{s(s)} + (m_b^2 + 2p_b \cdot k) g_4^{s(s)}],$$
$$f_7^{s(s)} = -\frac{1}{2} f_3^{s(s)}, \quad f_8^{s(s)} = -\frac{1}{2} f_4^{s(s)},$$
$$f_1^{b(s)} = \sum_{(i,j)} \xi_{(i,j)}^{(1)} \eta_{(i,j)}^{(1)} [2D_{27} - m_b^2(2D_{11} + D_{21}) - m_t^2 D_{23} - 2p_b \cdot k(D_{12} + D_{24}) + 2p_t \cdot k(D_{13} + D_{26}) + 2p_b \cdot p_t(D_{13} + D_{25})](-p_b, -k, p_t, m_i, m_b, m_b, m_j)$$
$$+ \frac{m_t m_b}{m_W} \sum_{i=H^0,h^0,G^0,A^0} \xi_i^{(3)} \{\eta^{(2)}[2m_b(-3D_{312} + (1 - \zeta_i)D_{27}) + m_b^3(D_0 + D_{12} - D_{22} - D_{32}) - m_t^2 m_b(D_{23} + 2D_{39}) - 2m_b p_b \cdot k(2D_{36} + D_{24} + \zeta_i(D_0 + D_{12})) + 2m_b p_t \cdot k(D_{25} + D_{310}) + 2m_b p_b \cdot p_t(D_{26} + 2D_{38})] + \eta^{(1)}[2m_t(-3D_{313} + (1 + \zeta_i)D_{27}) - m_t^3(D_{33} + (1 + \zeta_i)D_{23}) + m_b^2 m_t(D_{13} - 2D_{38} + (1 + \zeta_i)(D_0 - D_{22})) + 2m_t p_b \cdot k(D_{13} - D_{310} - (1 + \zeta_i)(D_{12} + D_{24})) + 2m_t p_t \cdot k(2D_{37} + (1 + \zeta_i)D_{25}) + 2m_t p_b \cdot p_t(2D_{39} + (1 + \zeta_i)D_{26})]\}(-k, -p_b, p_t, m_b, m_b, m_i, m_t),$$
$$f_2^{b(s)} = f_1^{b(s)}(\eta^{(1)} \leftrightarrow \eta^{(2)}, \eta_{(i,j)}^{(1)} \leftrightarrow \eta_{(i,j)}^{(2)}),$$
$$f_3^{b(s)} = \sum_{(i,j)} \xi_{(i,j)}^{(1)} \{\eta_{(i,j)}^{(2)} 2m_b \times [D_{11} + D_{21} + (1 + \zeta_i)(D_0 + D_{11})] - \eta_{(i,j)}^{(1)} 2m_t(D_{13} + D_{25})\} \times (-p_b, -k, p_t, m_i, m_b, m_b, m_j)$$
$$+ \frac{m_t m_b}{m_W} \sum_{i=H^0,h^0,G^0,A^0} \xi_i^{(3)} \{\eta^{(1)}[-4D_{27} + 2m_b^2(D_{22} - D_0 - (1 - \zeta_i)(D_{12} + D_{22})) + 2m_t^2(D_{23} - (1 + \zeta_i)D_{26}) + 4p_t \cdot k(D_{26} - D_{25})] + \eta^{(2)} 2m_t m_b(1 + \zeta_i)(D_{22} - D_{12} - D_{26})\}(-k, -p_b, p_t, m_b, m_b, m_i, m_t),$$
$$f_4^{b(s)} = f_3^{b(s)}(\eta^{(1)} \leftrightarrow \eta^{(2)}, \eta_{(i,j)}^{(1)} \leftrightarrow \eta_{(i,j)}^{(2)}),$$
$$f_5^{b(s)} = \sum_{(i,j)} \xi_{(i,j)}^{(1)} \{\eta_{(i,j)}^{(2)}(-2m_b)[D_{25} + (1 + \zeta_i)D_{13}] + \eta_{(i,j)}^{(1)} 2m_t D_{23}\}(-p_b, -k, p_t, m_i, m_b, m_b, m_j)$$
$$+ \frac{m_t m_b}{m_W} \sum_{i=H^0,h^0,G^0,A^0} \xi_i^{(3)} \{\eta^{(1)}[12D_{313} + 2m_b^2(2D_{38} - D_{13} + (1 - \zeta_i)(D_{13} + D_{26})) + 2m_t^2(D_{33} + (1 + \zeta_i)D_{23}) + 4p_b \cdot k(D_{25} + D_{310}) - 4p_t \cdot k(D_{23} + 2D_{37}) - 4p_t \cdot p_b(D_{23} + 2D_{39})] + \eta^{(2)} 2m_t m_b(1 + \zeta_i)(D_{13} + D_{23} - D_{26})\}(-k, -p_b, p_t, m_b, m_b, m_i, m_t),$$
$$f_6^{b(s)} = f_5^{b(s)}(\eta^{(1)} \leftrightarrow \eta^{(2)}, \eta_{(i,j)}^{(1)} \leftrightarrow \eta_{(i,j)}^{(2)}),$$

$$
\begin{aligned}
f_7^{b(s)} &= \sum_{(i,j)} \xi_{(i,j)}^{(1)} \{\eta_{(i,j)}^{(2)}(-m_b)[D_{11} + (1+\zeta_i)D_0] \\
&+ \eta_{(i,j)}^{(1)} m_t D_{13}\}(-p_b, -k, p_t, m_i, m_b, m_b, m_j) \\
&+ \frac{m_t m_b}{m_W} \sum_{i=H^0,h^0,G^0,A^0} \xi_i^{(3)} \{\eta^{(1)}[6(D_{27} - D_{311}) \\
&+ m_b^2(D_{11} - 2D_{12} - 2D_{22} - 2D_{36} \\
&+ (1+\zeta_i)(D_0 + D_{12})) \\
&- m_t^2(2D_{23} + 2D_{37} + (1+\zeta_i)D_{13}) \\
&- 2p_b\cdot k(D_{12} + 2D_{24} + 2D_{34}) \\
&+ 2p_t\cdot k(D_{13} + 2D_{25} + 2D_{35}) \\
&+ 2p_t\cdot p_b(D_{13} + 2D_{26} + D_{310})] \\
&+ \eta^{(2)} m_t m_b (1+\zeta_i)(D_{12} - D_{13} \\
&- D_0)\}(-k, -p_b, p_t, m_b, m_b, m_i, m_t), \\
f_8^{b(s)} &= f_7^{b(s)}(\eta^{(1)} \leftrightarrow \eta^{(2)}, \eta_{(i,j)}^{(1)} \leftrightarrow \eta_{(i,j)}^{(2)}), \\
f_9^{b(s)} &= \sum_{(i,j)} \xi_{(i,j)}^{(1)}[\eta_{(i,j)}^{(1)} \\
&\times 2(D_{12} + D_{24})](-p_b, -k, p_t, m_i, m_b, m_b, m_j) \\
&+ \frac{m_t m_b}{m_W} \sum_{i=H^0,h^0,G^0,A^0} \xi_i^{(3)} \{\eta^{(1)} 2m_t \\
&\times [-D_{13} - D_{26} + (1+\zeta_i)(D_{12} + D_{24})] \\
&+ \eta^{(2)} 2m_b[-D_{22} + D_{24} + \zeta_i(D_0 + 2D_{12} \\
&+ D_{24})]\}(-k, -p_b, p_t, m_b, m_b, m_i, m_t), \\
f_{10}^{b(s)} &= f_9^{b(s)}(\eta^{(1)} \leftrightarrow \eta^{(2)}, \eta_{(i,j)}^{(1)} \leftrightarrow \eta_{(i,j)}^{(2)}), \\
f_{11}^{b(s)} &= \sum_{(i,j)} \xi_{(i,j)}^{(1)}[-\eta_{(i,j)}^{(1)} \\
&\times 2(D_{13} + D_{26})](-p_b, -k, p_t, m_i, m_b, m_b, m_j) \\
&+ \frac{m_t m_b}{m_W} \\
&\times \sum_{i=H^0,h^0,G^0,A^0} \xi_i^{(3)} \{\eta^{(1)} 2m_t[D_{23} - (1+\zeta_i)D_{25}] \\
&- \eta^{(2)} 2m_b[-D_{26} + D_{25} + \zeta_i(D_{13} \\
&+ D_{25})]\}(-k, -p_b, p_t, m_b, m_b, m_i, m_t), \\
f_{12}^{b(s)} &= f_{11}^{b(s)}(\eta^{(1)} \leftrightarrow \eta^{(2)}, \eta_{(i,j)}^{(1)} \leftrightarrow \eta_{(i,j)}^{(2)}), \\
f_1^{V_1(t)} &= f_1^{V_1(s)}(U), \quad f_2^{V_1(t)} = f_2^{V_1(s)}(U), \\
f_5^{V_1(t)} &= f_4^{V_1(s)}(U), \quad f_6^{V_1(t)} = f_3^{V_1(s)}(U), \\
f_7^{V_1(t)} &= f_8^{V_1(s)}(U), \quad f_8^{V_1(t)} = f_7^{V_1(s)}(U), \\
f_{11}^{V_1(t)} &= -f_9^{V_1(s)}(U), \quad f_{12}^{V_1(t)} = -f_{10}^{V_1(s)}(U), \\
f_1^{s(t)} &= f_1^{s(s)}(U), \quad f_2^{s(t)} = f_2^{s(s)}(U), \\
f_5^{s(t)} &= f_4^{s(s)}(U), \quad f_6^{s(t)} = f_3^{s(s)}(U), \\
f_7^{s(t)} &= -\frac{1}{2} f_5^{s(s)}(U), \quad f_8^{s(t)} = -\frac{1}{2} f_6^{s(s)}(U), \\
f_1^{b(t)} &= f_1^{b(s)}(U), \quad f_2^{b(t)} = f_2^{b(s)}(U), \\
f_3^{b(t)} &= f_6^{b(s)}(U), \quad f_4^{b(t)} = f_5^{b(s)}(U), \\
f_5^{b(t)} &= f_4^{b(s)}(U), \quad f_6^{b(t)} = f_3^{b(s)}(U), \\
f_7^{b(t)} &= f_8^{b(s)}(U), \quad f_8^{b(t)} = f_7^{b(s)}(U), \\
f_9^{b(t)} &= -f_{11}^{b(s)}(U), \quad f_{10}^{b(t)} = -f_{12}^{b(s)}(U), \\
f_{11}^{b(t)} &= -f_9^{b(s)}(U), \quad f_{12}^{b(t)} = -f_{10}^{b(s)}(U).
\end{aligned}
$$

Here the sums over (i,j) run over (H^0, H^-), (h^0, H^-), (H^0, G^-), (h^0, G^-) and (A^0, G^-), and U is a transformation defined by

$$
\begin{aligned}
&p_b \to p_t, \quad \hat{s} \to \hat{t}, \quad k \to -k, \\
&\xi_i^{(1)} \to \xi_i^{(2)}, \quad \xi_i^{(3)} \to \xi_i^{(4)}, \quad m_t \leftrightarrow m_b, \\
&\eta^{(1)} \leftrightarrow \eta^{(2)}, \quad \eta_{(i,j)}^{(1)} \leftrightarrow \eta_{(i,j)}^{(2)}, \quad \xi_{(i,j)}^{(1)} \leftrightarrow \xi_{(i,j)}^{(2)},
\end{aligned}
$$

and D_0, D_{ij}, D_{ijk} are the four-point Feynman integrals [18]. All other form factors f_i^l not listed above vanish. In the above expressions we have used the following definitions:

$$
\begin{aligned}
\eta_{(H^0,H^-)}^{(1)} &= \eta_{(h^0,H^-)}^{(1)} = \eta^{(1)} = m_b \tan\beta, \\
\eta_{(H^0,G^-)}^{(1)} &= \eta_{(h^0,G^-)}^{(1)} = -\eta_{(A^0,G^-)}^{(1)} = -m_b, \\
\eta_{(H^0,H^-)}^{(2)} &= \eta_{(h^0,H^-)}^{(2)} = \eta^{(2)} = m_t \cot\beta, \\
\eta_{(H^0,G^-)}^{(2)} &= \eta_{(h^0,G^-)}^{(2)} = \eta_{(A^0,G^-)}^{(2)} = m_t, \\
\xi_{H^0}^{(1)} &= \frac{\cos^2\alpha}{\cos^2\beta}, \quad \xi_{h^0}^{(1)} = \frac{\sin^2\alpha}{\cos^2\beta}, \\
\xi_{A^0}^{(1)} &= \tan^2\beta, \quad \xi_{G^0}^{(1)} = 1, \\
\xi_{H^0}^{(2)} &= \frac{\sin^2\alpha}{\sin^2\beta}, \quad \xi_{h^0}^{(2)} = \frac{\cos^2\alpha}{\sin^2\beta}, \\
\xi_{A^0}^{(2)} &= \cot^2\beta, \quad \xi_{G^0}^{(2)} = 1, \\
\xi_{H^0}^{(3)} &= -\xi_{h^0}^{(3)} = \frac{\sin\alpha\cos\alpha}{\sin\beta\cos\beta}, \\
\xi_{G^0}^{(3)} &= -\xi_{A^0}^{(3)} = 1, \quad \xi_{H^-}^{(1)} = \frac{m_t^2}{m_b^2}\cot^2\beta, \\
\xi_{G^-}^{(1)} &= \frac{m_t^2}{m_b^2}, \quad \xi_{H^-}^{(2)} = \frac{m_b^2}{m_t^2}\tan^2\beta, \\
\xi_{G^-}^{(2)} &= \frac{m_b^2}{m_t^2}, \quad \xi_{H^-}^{(3)} = \tan^2\beta, \quad \xi_{G^-}^{(3)} = 1, \\
\xi_{H^-}^{(4)} &= \cot^2\beta, \quad \xi_{G^-}^{(4)} = 1,
\end{aligned}
$$

$$
\begin{aligned}
\xi_{(H^0,H^-)}^{(1)} &= 2m_b\frac{\cos\alpha}{\cos\beta}\Big[m_W\cos(\beta-\alpha) \\
&- \frac{m_Z}{2\cos\theta_W}\cos 2\beta\cos(\beta+\alpha)\Big], \\
\xi_{(h^0,H^-)}^{(1)} &= -2m_b\frac{\sin\alpha}{\cos\beta}\Big[m_W\sin(\beta-\alpha) \\
&+ \frac{m_Z}{2\cos\theta_W}\cos 2\beta\sin(\beta+\alpha)\Big], \\
\xi_{(H^0,G^-)}^{(1)} &= m_b\frac{\cos\alpha}{\cos\beta}\Big[m_W\sin(\beta-\alpha) \\
&- \frac{m_Z}{\cos\theta_W}\sin 2\beta\cos(\beta+\alpha)\Big],
\end{aligned}
$$

$$\xi^{(1)}_{(h^0,G^-)} = m_b \frac{\sin\alpha}{\cos\beta}\bigg[m_W \cos(\beta-\alpha) - \frac{m_Z}{\cos\theta_W}\sin 2\beta \sin(\beta+\alpha)\bigg],$$

$$\xi^{(1)}_{(A^0,G^-)} = m_b m_W \tan\beta,$$

$$\xi^{(2)}_{(H^0,H^-)} = 2m_t \frac{\sin\alpha}{\sin\beta}\bigg[m_W \cos(\beta-\alpha) - \frac{m_Z}{2\cos\theta_W}\cos 2\beta \cos(\beta+\alpha)\bigg],$$

$$\xi^{(2)}_{(h^0,H^-)} = 2m_t \frac{\cos\alpha}{\sin\beta}\bigg[m_W \sin(\beta-\alpha) + \frac{m_Z}{2\cos\theta_W}\cos 2\beta \sin(\beta+\alpha)\bigg],$$

$$\xi^{(2)}_{(H^0,G^-)} = m_t \frac{\sin\alpha}{\sin\beta}\bigg[m_W \sin(\beta-\alpha) - \frac{m_Z}{\cos\theta_W}\sin 2\beta \cos(\beta+\alpha)\bigg],$$

$$\xi^{(2)}_{(h^0,G^-)} = -m_t \frac{\cos\alpha}{\sin\beta}\bigg[m_W \cos(\beta-\alpha) - \frac{m_Z}{\cos\theta_W}\sin 2\beta \sin(\beta+\alpha)\bigg],$$

$$\xi^{(2)}_{(A^0,G^-)} = m_t m_W \cot\beta,$$

$$\zeta_{H^0} = \zeta_{h^0} = \zeta_{H^-} = -\zeta_{A^0} = -\zeta_{G^0} = -\zeta_{G^-} = 1,$$

$$g_1^{V_1(s)} = \sum_{i=H^0,h^0,G^0,A^0} \xi_i^{(1)}\bigg\{\bigg[\frac{1}{2} - 2\overline{C}_{24} + m_b^2(-2C_{11} + C_{12} - C_{21} + C_{23}) - \hat{s}(C_{12} + C_{23})\bigg](-p_b, -k, m_i, m_b, m_b) + [-F_0 + F_1 + 2m_b^2 G_1 - (1+\zeta_i)2m_b^2 G_0](m_b^2, m_i, m_b)\bigg\},$$

$$g_2^{V_1(s)} = \sum_{i=H^-,G^-} 2\bigg\{\xi_i^{(1)}\bigg[\frac{1}{2} - 2\overline{C}_{24} + m_t^2 C_0 + m_b^2(-C_0 - 2C_{11} + C_{12} - C_{21} + C_{23}) - \hat{s}(C_{12} + C_{23})\bigg](-p_b, -k, m_i, m_t, m_t) + [\xi_i^{(1)}(-F_0 + F_1) - 2m_t^2 \zeta_i G_0 + m_b^2(\xi_i^{(1)} + \xi_i^{(3)})(G_1 - \zeta_i G_0)](m_b^2, m_i, m_t)\bigg\},$$

$$g_3^{V_1(s)} = g_2^{V_1(s)}(\xi_i^{(1)} \leftrightarrow \xi_i^{(3)};\quad i = H^-, G^-),$$

$$g_4^{V_1(s)} = \sum_{i=H^0,h^0,G^0,A^0} \xi_i^{(1)} 2m_b[C_0 + 2C_{11} + C_{21} + \zeta_i(C_0 + C_{11})](-p_b, -k, m_i, m_b, m_b) + \sum_{i=H^-,G^-} 4m_b\bigg[\xi_i^{(3)}(C_0 + 2C_{11} + C_{21}) + \frac{m_t^2}{m_b^2}\zeta_i(C_0 + C_{11})\bigg](-p_b, -k, m_i, m_t, m_t),$$

$$g_5^{V_1(s)} = g_4^{V_1(s)}(\xi_i^{(1)} \leftrightarrow \xi_i^{(3)}; i = H^-, G^-),$$

$$g_6^{V_1(s)} = -\sum_{i=H^0,h^0,G^0,A^0} \xi_i^{(1)} m_b(C_0 + C_{11} + \zeta_i C_0)(-p_b, -k, m_i, m_b, m_b) - \sum_{i=H^-,G^-} 2m_b\bigg[\xi_i^{(3)}(C_0 + C_{11}) + \frac{m_t^2}{m_b^2}\zeta_i C_0\bigg](-p_b, -k, m_i, m_t, m_t),$$

$$g_7^{V_1(s)} = g_6^{V_1(s)}(\xi_i^{(1)} \leftrightarrow \xi_i^{(3)}; i = H^-, G^-),$$

$$g_8^{V_1(s)} = \sum_{i=H^0,h^0,G^0,A^0} \xi_i^{(1)} 2(C_{12} + C_{23})(-p_b, -k, m_i, m_b, m_b) + \sum_{i=H^-,G^-} 4\xi_i^{(1)}(C_{12} + C_{24})(-p_b, -k, m_i, m_t, m_t),$$

$$g_9^{V_1(s)} = g_8^{V_1(s)}(\xi_i^{(1)} \leftrightarrow \xi_i^{(3)}; i = H^-, G^-),$$

$$g_1^{V_2(s)} = \sum_{i=H^0,h^0,G^0,A^0} \frac{m_t m_b}{m_W^2}\xi_i^{(3)}\bigg\{\eta^{(1)}\bigg[-\frac{1}{2} + 4\overline{C}_{24} + m_t^2(C_0 + 2C_{11} + \zeta_i(C_0 + C_{11}) + C_{21} - C_{12} - C_{23}) + m_{H^-}^2(C_{22} - C_{23}) + \hat{s}(C_{12} + C_{23})\bigg] + \eta^{(2)} m_b m_t[C_0 + \zeta_i(C_0 + C_{11})]\bigg\}(-p_t, -p_{H^-}, m_i, m_t, m_b) + \sum_{(i,j)} \frac{1}{m_W}\{\xi^{(2)}_{(i,j)}\eta^{(2)}_{(i,j)} m_t[(1+\zeta_i)C_0 + C_{12}](-p_{H^-}, -p_t, m_j, m_i, m_t) + \xi^{(1)}_{(i,j)}[\eta^{(1)}_{(i,j)} m_t(C_0 + C_{12}) + \eta^{(2)}_{(i,j)} m_b \zeta_i C_0](-p_{H^-}, -p_t, m_i, m_j, m_b)\},$$

$$g_2^{V_2(s)} = g_1^{V_2(s)}(\eta^{(1)} \leftrightarrow \eta^{(2)}, \eta^{(1)}_{(i,j)} \leftrightarrow \eta^{(2)}_{(i,j)}),$$

$$g_3^{V_2(s)} = \sum_{i=H^0,h^0,G^0,A^0} \frac{m_t m_b}{m_W^2}\xi_i^{(3)}\{\eta^{(1)} m_t[C_0 + C_{11} + \zeta_i(C_0 + C_{12})] + \eta^{(2)}\zeta_i m_b C_{12}\}(-p_t, -p_{H^-}, m_i, m_t, m_b) + \sum_{(i,j)} \frac{1}{m_W}[\xi^{(2)}_{(i,j)}\eta^{(2)}_{(i,j)}(C_0 + C_{11})(-p_{H^-}, -p_t, m_j, m_i, m_t) + \xi^{(1)}_{(i,j)}\eta^{(1)}_{(i,j)}(C_0 + C_{11})(-p_{H^-}, -p_t, m_i, m_j, m_b)],$$

$$g_4^{V_2(s)} = g_3^{V_2(s)}(\eta^{(1)} \leftrightarrow \eta^{(2)}, \eta^{(1)}_{(i,j)} \leftrightarrow \eta^{(2)}_{(i,j)}),$$

$$
\begin{aligned}
g_1^{V_2(t)} &= \sum_{i=H^0,h^0,G^0,A^0} \frac{m_t m_b}{m_W^2}\xi_i^{(3)}\Big\{\eta^{(1)}\Big[-\frac{1}{2}+4\overline{C}_{24} \\
&+ m_b^2(C_0+2C_{11}+\zeta_i(C_0+C_{11}) \\
&+ C_{21}-C_{12}-C_{23})+m_{H^-}^2(C_{22}-C_{23}) \\
&+ \hat{t}(C_{12}+C_{23})\Big]+\eta^{(2)}m_b m_t[C_0+\zeta_i(C_0 \\
&+ C_{11})]\Big\}(-p_b,p_{H^-},m_i,m_b,m_t) \\
&+ \sum_{(i,j)}\frac{1}{m_W}\{\xi_{(i,j)}^{(1)}\eta_{(i,j)}^{(2)}m_t[(1+\zeta_i)C_0 \\
&+ C_{12}](-p_{H^-},p_b,m_j,m_i,m_b) \\
&+ \xi_{(i,j)}^{(2)}[\eta_{(i,j)}^{(1)}m_b(C_0+C_{12}) \\
&+ \eta_{(i,j)}^{(2)}m_t\zeta_i C_0](-p_{H^-},p_b,m_i,m_j,m_t)\}, \\
g_2^{V_2(t)} &= g_1^{V_2(t)}(\eta^{(1)}\leftrightarrow\eta^{(2)},\eta_{(i,j)}^{(1)}\leftrightarrow\eta_{(i,j)}^{(2)}), \\
g_3^{V_2(t)} &= \sum_{i=H^0,h^0,G^0,A^0} -\frac{m_t m_b}{m_W^2}\xi_i^{(3)}\{\eta^{(1)}m_b[C_0+C_{11} \\
&+ \zeta_i(C_0+C_{12})] \\
&+ \eta^{(2)}\zeta_i m_t C_{12}\}(-p_b,p_{H^-},m_i,m_b,m_t) \\
&- \sum_{(i,j)}\frac{1}{m_W}[\xi_{(i,j)}^{(1)}\eta_{(i,j)}^{(2)}(C_0+C_{11}) \\
&\times (-p_{H^-},p_b,m_j,m_i,m_t) \\
&+ \xi_{(i,j)}^{(2)}\eta_{(i,j)}^{(1)}(C_0+C_{11})(-p_{H^-},p_b,m_i,m_j,m_t)], \\
g_4^{V_2(t)} &= g_3^{V_2(t)}(\eta^{(1)}\leftrightarrow\eta^{(2)},\eta_{(i,j)}^{(1)}\leftrightarrow\eta_{(i,j)}^{(2)}), \\
g_1^{s(s)} &= \sum_{i=H^0,h^0,G^0,A^0} m_b\xi_i^{(1)}\{-\zeta_i F_0(p_b+k,m_i,m_b) \\
&+ [\zeta_i F_0-2m_b^2(1+\zeta_i)G_0+2m_b^2G_1](m_b^2,m_i,m_b)\} \\
&+ \sum_{i=H^-,G^-} 2m_b\Big\{-\frac{m_t^2}{m_b^2}\zeta_i F_0(p_b+k,m_i,m_t) \\
&+ \Big[-2m_t^2\zeta_i G_0+m_b^2(\xi_i^{(1)}+\xi_i^{(3)})(G_1-\zeta_i G_0) \\
&+ \zeta_i\frac{m_t^2}{m_b^2}F_0\Big](m_b^2,m_i,m_t)\Big\}, \\
g_2^{s(s)} &= \sum_{i=H^0,h^0,G^0,A^0} \xi_i^{(1)}(-F_0+F_1)(p_b+k,m_i,m_b), \\
g_3^{s(s)} &= \sum_{i=H^0,h^0,G^0,A^0} \xi_i^{(1)}[F_0-F_1-2m_b^2G_1 \\
&+ 2(1+\zeta_i)m_b^2G_0](m_b^2,m_i,m_b) \\
&+ \sum_{i=H^-,G^-} 2\{\xi_i^{(1)}(-F_0+F_1)(p_b+k,m_i,m_t) \\
&- [\xi_i^{(1)}(-F_0+F_1)-2\zeta_i m_t^2G_0+m_b^2(\xi_i^{(1)}+\xi_i^{(3)}) \\
&\times (G_1-\zeta_i G_0)](m_b^2,m_i,m_t)\}, \\
g_4^{s(s)} &= g_3^{s(s)}(\xi_i^{(1)}\leftrightarrow\xi_i^{(3)};i=H^-,G^-),
\end{aligned}
$$

$$
\begin{aligned}
\delta\Lambda_{\mathrm{L}}^{btH^-} &= \frac{4N_c}{3m_W^2}(1-\cot^2\theta_W)\Big[2m_t^2\Big(\ln\frac{m_t^2}{\mu^2}-1\Big) \\
&+ m_b^2+m_t^2-\frac{5}{6}m_W^2+m_b^2F_0 \\
&+ (m_b^2-m_t^2-2m_W^2)F_1\Big](m_W^2,m_b,m_t) \\
&+ \frac{4N_c}{3m_W^2}\cot^2\theta_W\Big\{-\frac{5}{6}[(g_V^b)^2+(g_A^b)^2 \\
&+ (g_V^t)^2+(g_A^t)^2]m_Z^2 \\
&+ \Big[((g_V^t)^2+(g_A^t)^2)(2m_t^2\ln\frac{m_t^2}{\mu^2}+m_t^2F_0 \\
&- 2m_Z^2F_1) \\
&- ((g_V^t)^2-(g_A^t)^2)3m_t^2F_0\Big](m_Z^2,m_t,m_t) \\
&+ \Big[((g_V^b)^2+(g_A^b)^2)\Big(2m_b^2\ln\frac{m_b^2}{\mu^2}+m_b^2F_0 \\
&- 2m_Z^2F_1\Big)-((g_V^b)^2 \\
&- (g_A^b)^2)3m_b^2F_0\Big](m_Z^2,m_b,m_b)\Big\} \\
&+ \frac{4N_c}{m_W^2}[(\cot^2\beta-1)m_t^2F_0 \\
&+ (m_t^2-m_b^2-2m_t^2\cot^2\beta)F_1 \\
&+ (m_t^2\cot^2\beta+m_b^2\tan^2\beta+2m_b^2)m_t^2G_0 \\
&- (m_t^2\cot^2\beta+m_b^2\tan^2\beta)m_{H^-}^2G_1](m_{H^-}^2,m_t,m_b) \\
&+ \sum_{i=H^0,h^0,G^0,A^0}\frac{1}{2m_W^2}\{m_b^2\xi_i^{(1)}[F_1-F_0 \\
&- 2m_b^2(1+\zeta_i)G_0+2m_b^2G_1](m_b^2,m_i,m_b) \\
&- m_t^2\xi_i^{(2)}[-F_0+F_1-2\zeta_i F_0+2m_t^2(1+\zeta_i)G_0 \\
&- 2m_t^2G_1](m_t^2,m_i,m_t)\} \\
&+ \sum_{i=H^-,G^-}\frac{1}{m_W^2}\Big\{m_b^2[\xi_i^{(1)}(-F_0+F_1)-2m_t^2\zeta_i G_0 \\
&+ m_b^2(\xi_i^{(1)}+\xi_i^{(3)})(G_1-\zeta_i G_0)](m_b^2,m_i,m_t) \\
&- m_t^2\Big[-\frac{2m_b^2}{m_t^2}\zeta_i F_0+\xi_i^{(2)}(-F_0+F_1)+2m_b^2\zeta_i G_0 \\
&- m_t^2(\xi_i^{(2)}+\xi_i^{(4)})(G_1-\zeta_i G_0)\Big](m_t^2,m_i,m_b)\Big\}, \\
\delta\Lambda_R^{btH^-} &= \sum_{i=H^0,h^0,G^0,A^0}\frac{1}{2m_W^2}\{m_t^2\xi_i^{(2)}[-F_0+F_1 \\
&- 2m_t^2(1+\zeta_i)G_0+2m_t^2G_1](m_t^2,m_i,m_t) \\
&- m_b^2\xi_i^{(1)}[-F_0+F_1-2\zeta_i F_0+2m_b^2(1+\zeta_i)G_0 \\
&- 2m_b^2G_1](m_b^2,m_i,m_b)\}
\end{aligned}
$$

$$+\sum_{i=H^-,G^-}\frac{1}{m_W^2}\Bigg\{m_t^2[\xi_i^{(2)}(-F_0+F_1)-2m_b^2\zeta_i G_0 + m_t^2(\xi_i^{(2)}+\xi_i^{(4)})(G_1-\zeta_i G_0)](m_t^2,m_i,m_b) - m_b^2\Bigg[-\frac{2m_t^2}{m_b^2}\zeta_i F_0+\xi_i^{(1)}(-F_0+F_1) + 2m_t^2\zeta_i G_0 - m_b^2(\xi_i^{(1)}+\xi_i^{(3)}) \times (G_1-\zeta_i G_0)\Bigg](m_b^2,m_i,m_t)\Bigg\}.$$

Here C_0, C_{ij} are the three-point Feynman integrals [18] and $\overline{C}_{24} \equiv -(1/4)\Delta + C_{24}$, while

$$F_n(q,m_1,m_2) = \int_0^1 \mathrm{d}y y^n \times \ln\left[\frac{-q^2y(1-y)+m_1^2(1-y)+m_2^2y}{\mu^2}\right],$$

$$G_n(q,m_1,m_2) = -\int_0^1 \mathrm{d}y \times \frac{y^{n+1}(1-y)}{-q^2y(1-y)+m_1^2(1-y)+m_2^2y},$$

and

$$g_V^t = \frac{1}{2}-\frac{4}{3}\sin^2\theta_W, \quad g_A^t = \frac{1}{2},$$
$$g_V^b = -\frac{1}{2}+\frac{2}{3}\sin^2\theta_W, \quad g_A^b = -\frac{1}{2},$$

which are the SM couplings of the top and bottom quarks to the Z boson.

Appendix B

$$h_1^{(s)} = 4m_t^2\cot\beta(2p_b\cdot k - m_b^2) - 4m_b^2\tan\beta(p_b\cdot p_t + p_t\cdot k),$$
$$h_2^{(s)} = -4m_bm_t\cot\beta(p_b\cdot p_t + p_t\cdot k) + 4m_bm_t\tan\beta(2p_b\cdot k - m_b^2),$$
$$h_3^{(s)} = 2m_t\cot\beta(2p_b\cdot kp_b\cdot p_t - m_b^2p_t\cdot k - 2m_b^2p_b\cdot p_t) + 2m_b^2m_t\tan\beta(p_b\cdot k - 2m_b^2),$$
$$h_4^{(s)} = 2m_t^2m_b\cot\beta(p_b\cdot k - 2m_b^2) + 2m_b\tan\beta(2p_b\cdot kp_b\cdot p_t - m_b^2p_t\cdot k - 2m_b^2p_b\cdot p_t),$$
$$h_5^{(s)} = 2m_t\cot\beta(m_t^2p_b\cdot k - 2(p_b\cdot p_t)^2) + 2m_b^2m_t\tan\beta(p_t\cdot k - 2p_b\cdot p_t),$$
$$h_6^{(s)} = 2m_t^2m_b\cot\beta(p_t\cdot k - 2p_b\cdot p_t) + 2m_b\tan\beta(m_t^2p_b\cdot k - 2(p_b\cdot p_t)^2),$$
$$h_7^{(s)} = 4m_t\cot\beta(m_b^2p_t\cdot k - 2p_b\cdot kp_b\cdot p_t - 2p_b\cdot kp_t\cdot k) - 4m_b^2m_t\tan\beta p_b\cdot k,$$
$$h_8^{(s)} = -4m_t^2m_b\cot\beta p_b\cdot k + 4m_b\tan\beta(m_b^2p_t\cdot k - 2p_b\cdot kp_b\cdot p_t - 2p_b\cdot kp_t\cdot k),$$
$$h_9^{(s)} = 4m_t^2\cot\beta p_b\cdot k(p_b\cdot k - m_b^2) - 4m_b^4\tan\beta p_t\cdot k,$$
$$h_{10}^{(s)} = -4m_b^3m_t\cot\beta p_t\cdot k + 4m_bm_t\tan\beta p_b\cdot k(p_b\cdot k - m_b^2),$$
$$h_{11}^{(s)} = 4m_t^2\cot\beta p_b\cdot k(p_t\cdot k - p_b\cdot p_t) - 4m_b^2\tan\beta p_t\cdot kp_b\cdot p_t,$$
$$h_{12}^{(s)} = -4m_bm_t\cot\beta p_t\cdot kp_b\cdot p_t + 4m_bm_t\tan\beta p_b\cdot k(p_t\cdot k - p_b\cdot p_t),$$
$$h_1^{(t)} = 4m_t^2\cot\beta(2p_b\cdot k - p_b\cdot p_t) - 4m_b^2\tan\beta(m_t^2 + p_t\cdot k),$$
$$h_2^{(t)} = -4m_bm_t\cot\beta(m_t^2 + p_t\cdot k) + 4m_bm_t\tan\beta(2p_b\cdot k - p_b\cdot p_t),$$
$$h_3^{(t)} = 2m_t\cot\beta(2p_b\cdot kp_b\cdot p_t - m_b^2p_t\cdot k - 2(p_b\cdot p_t)^2) + 2m_b^2m_t\tan\beta(p_b\cdot k - 2p_b\cdot p_t),$$
$$h_4^{(t)} = 2m_t^2m_b\cot\beta(p_b\cdot k - 2p_b\cdot p_t) + 2m_b\tan\beta(2p_b\cdot kp_b\cdot p_t - m_b^2p_t\cdot k - 2(p_b\cdot p_t)^2),$$
$$h_5^{(t)} = 2m_t^3\cot\beta(p_b\cdot k - 2p_b\cdot p_t) + 2m_b^2m_t\tan\beta(p_t\cdot k - 2m_t^2),$$
$$h_6^{(t)} = 2m_t^2m_b\cot\beta(p_t\cdot k - 2m_t^2) + 2m_bm_t^2\tan\beta(p_b\cdot k - 2p_b\cdot p_t),$$
$$h_7^{(t)} = -4m_t\cot\beta(m_t^2p_b\cdot k + 2p_b\cdot kp_t\cdot k) - 4m_b^2m_t\tan\beta p_t\cdot k,$$
$$h_8^{(t)} = -4m_t^2m_b\cot\beta p_t\cdot k - 4m_b\tan\beta(m_t^2p_b\cdot k + 2p_b\cdot kp_t\cdot k),$$
$$h_9^{(t)} = 4m_t^2\cot\beta p_b\cdot k(p_b\cdot k - p_b\cdot p_t) - 4m_b^2\tan\beta p_b\cdot p_tp_t\cdot k,$$
$$h_{10}^{(t)} = -4m_bm_t\cot\beta p_b\cdot p_tp_t\cdot k + 4m_bm_t\tan\beta p_b\cdot k(p_b\cdot k - p_b\cdot p_t),$$
$$h_{11}^{(t)} = 4m_t^2\cot\beta p_b\cdot k(p_t\cdot k - m_t^2) - 4m_b^2m_t^2\tan\beta p_t\cdot k,$$
$$h_{12}^{(t)} = -4m_bm_t^3\cot\beta p_t\cdot k + 4m_bm_t\tan\beta p_b\cdot k(p_t\cdot k - m_t^2).$$

References

1. For a review, see J. Gunion, H. Haber, G. Kane, S. Dawson, The Higgs Hunter's Guide (Addison-Wesley, New York 1990)
2. H.E. Haber, G.L. Kane, Phys. Rep. **117**, 75 (1985); J.F. Gunion, H.E. Haber, Nucl. Phys. B **272**, 1 (1986)

3. E. Eichten, I. Hinchliffe, K. Lane, C. Quigg, Rev. Mod. Phys. **56**, 579 (1984); 1065 (E) (1986); N.G. Deshpande, X. Tata, D.A. Dicus, Phys. Rev. D **29**, 1527 (1984); S.S.D. Willenbrock, Phys. Rev. D **35**, 173 (1987); A. Krause, T. Plehn, M. Spria, P.M. Zerwas, Nucl. Phys. B **519**, 85 (1998); J. Yi, M. Wen-Gan, H. Liang, H. Meng, Y. Zeng-Hui, J. Phys. G **23**, 385 (1997); Erratum ibid. G **23**, 1151 (1997)
4. D.A. Dicus, J.L. Hewett, C. Kao, T.G. Rizzo, Phys. Rev. D **40**, 787 (1989); A.A. Barrientos Bendezú, B.A. Kniehl, Phys. Rev. D 59, 015009-1 (1999)
5. S. Moretti, K. Odagiri, Phys. Rev. D **59**, 055008-1 (1999)
6. Z. Kunszt, F. Zwirner, Nucl. Phys. B **385**, 3 (1992) and references cited therein
7. J.F. Gunion, H.E. Haber, F.E. Paige, W.-K. Tung, S.S.D. Willenbrock, Nucl. Phys. B **294**, 621 (1987); R.M. Barnett, H.E. Haber, D.E. Soper, ibid. B **306**, 697 (1988); F.I. Olness, W.-K. Tung, ibid. B **308**, 813 (1988)
8. V. Barger, R.J.N. Phillips, D.P. Roy, Phys. Lett. B **324**, 236 (1994)
9. C.S. Huang, S.H. Zhu, Phys. Rev. D **60**, 075012 (1999)
10. K. Odagiri, hep-ph/9901432; Phys. Lett. B **452**, 327 (1999)
11. D.P. Roy, Phys. Lett. B **459**, 607 (1999)
12. Francesca Borzumati, Jean-Loic Kneur, Nir Polonsky, Phys. Rev. D **60**, 115011 (1999)
13. S. Sirlin, Phys. Rev. D **22**, 971 (1980); W.J. Marciano, A. Sirlin, ibid. **22**, 2695 (1980); 31, 213 (E) (1985); A. Sirlin, W.J. Marciano, Nucl. Phys. B **189**, 442 (1981); K.I. Aoki et al., Prog. Theor. Phys. Suppl. **73**, 1 (1982)
14. A. Mendez, A. Pomarol, Phys. Lett. B **279**, 98 (1992)
15. Particle Data Group, C. Caso et al., Eur. Phys. J. C **3**, 1 (1998)
16. J. Gunion, A. Turski, Phys. Rev. D **39**, 2701 (1989); D **40**, 2333 (1990); J.R. Espinosa, M. Quiros, Phys. Lett. B **266**, 389 (1991); M. Carena, M. Quiros, C.E.M. Wagner, Nucl. Phys. B **461**, 407 (1996)
17. CTEQ collaboration (H.L. Lai et al.), hep-ph/9903282
18. G. Passarino, M. Veltman, Nucl. Phys. B **160**, 151 (1979); A. Axelrod, ibid. B **209**, 349 (1982); M. Clements et al., Phys. Rev. D **27**, 570 (1983)